

TRIADIC SPECTRAL LAYERING

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Abstract— This study develops **Triadic Spectral Layering** (TSL) as a rigorous compositional methodology in which each pitch of a chord is understood not as a discrete atom but as a generator of an acoustic projection — a set of harmonically implied pitches derived from its overtone series or a selected modal subset thereof. The chord thus becomes a **harmonic field**: a superposition of interacting projections that the composer may selectively activate, suppress, or continuously transform through microtonal inflection. The work proceeds from acoustic first principles (the harmonic series, just-intonation deviations, and their equal-temperament approximations), through a formal algebraic specification of the TSL operation, to analytical case studies on specific harmonic objects, and finally to a fully articulated compositional methodology with extended worked examples. Historical practice in Debussy, Ravel, Messiaen, Grisey, and Murail is treated analytically — as anticipations or implementations of the same underlying logic — rather than as mere biographical context. The primary contribution is the **composition-facing formalism**: a set of defined operations (projection, union, reduction, weighting, register distribution, spectral smoothing) that a composer can apply systematically to generate, transform, and voice-lead between harmonic fields of controlled density and color. **Keywords**: harmonic field, overtone series, spectral composition, triadic projection, modal interference, microtonality, voice-leading geometry, Triadic Spectral Layering.

INTRODUCTION: FROM CHORD TO FIELD

The Limitations of Functional Harmony

Western tonal harmony, as codified in the treatises of Rameau, Riemann, and their successors, rests upon two foundational assumptions that, while enormously productive, also constitute structural limitations for the composer seeking to work beyond the tonal canon. The first assumption is **discreteness**: pitches are drawn from a closed, finite set of twelve equal-tempered classes; chords are unordered subsets of this set; and harmonic relationships are defined by interval-class membership alone. The acoustic reality of sound — its continuous spectrum, its dependence on register and timbre, its microtonal shading — is erased from the theoretical model. The second assumption is **directionality**: chords acquire meaning principally through their function within a tonal trajectory — as tonic, dominant, subdominant, or their chromatic substitutes. A chord has, in this framework, no intrinsic acoustic identity independent of its functional role; it is a relational object, not an autonomous one. These assumptions generate powerful consequences: voice-leading principles, chord-scale theory, the logic of the ii–V–I progression, the entire architecture of common-practice modulation. But they also foreclose whole domains of harmonic thought:

- Chords that do not participate in functional progressions are theorized poorly or not at all.
- The interior acoustic structure of a chord (which partials reinforce, which clash, which register a tone implies) is invisible.
- Microtonality — inflection between equal-tempered pitches — has no place in the model.
- Register is treated as a performance detail rather than a structural parameter.

The Limitations of Strict Modal Systems

Modal composition, from medieval polyphony through twentieth-century neo-modalism, addresses some of these limitations by loosening directionality. A mode is a pitch collection with a characteristic hierarchy (final, dominant reciting tone, half-step positions), and chords built within it need not progress functionally. Yet strict modal systems retain the assumption of discreteness and add a new limitation: the **scalar prison**. Every pitch must belong to the mode; vertical combinations are validated by their scalar membership rather than by their acoustic logic. The result is harmonic language that is often beautiful but rarely spectrally coherent: the chords are not derived from the acoustic properties of their constituent pitches but from the arbitrary intersections of a scale template.

The Harmonic Field

The concept of the **harmonic field** — developed most fully in the spectralist tradition but implicit already in Debussy's parallelism and Messiaen's chord-color theory — replaces the chord-as-function with the **chord-as-acoustic-object**. A harmonic field is a region of pitch-space activated by the resonance properties of a generating sonority. It is not a fixed set of pitches but a **probability distribution** over pitch-space, in which some pitches are strongly implied (low partials of the overtone series), others weakly implied (high or deviant partials), and the composer's task is to select, weight, and voice-lead among these implications. This reconceptualization has three immediate consequences:

1. **Register becomes structural**: the harmonic series is not transposition-invariant; what a pitch generates in the bass register differs acoustically from what it generates in the treble.
2. **Microtonality becomes compositionally necessary**: the natural harmonics of any pitch deviate from equal temperament by amounts that are musically significant, and ignoring these deviations coarsens the harmonic palette.
3. **Chords become fields**: a three-note chord generates not a triad but a superposition of three independent overtone projections, each rich in implied content.

Situating Triadic Spectral Layering

Triadic Spectral Layering (TSL) is the compositional formalization of this field concept, specialized to the case in which the projection of each pitch is organized as a **triadic** or **modal** subset of that pitch's overtone series. The name reflects the method's roots: it begins with the triad — the simplest acoustically grounded harmonic object — and extends outward through layering (superposition of projections) toward the full spectral continuum. TSL occupies a position between:

Partial n	Ratio $n : 1$	Interval Name	ET Pitch (C1 root)	Just Cents	ET Cents / Deviation
1	1:1	Unison	C1	0	0 / ± 0
2	2:1	Octave	C2	1200	1200 / ± 0
3	3:1	Octave + P5	G2	1902	1900 / +2
4	4:1	2 Octaves	C3	2400	2400 / ± 0
5	5:1	2 Oct + M3	E3	2786	2800 / -14
6	6:1	2 Oct + P5	G3	3102	3100 / +2
7	7:1	2 Oct + m7↓	B♭3↓	3369	3400 / -31
8	8:1	3 Octaves	C4	3600	3600 / ± 0
9	9:1	3 Oct + M2	D4	3804	3800 / +4
10	10:1	3 Oct + M3	E4	3986	4000 / -14
11	11:1	3 Oct + aug4↑	F#4↑	4151	4100 / +51
12	12:1	3 Oct + P5	G4	4302	4300 / +2
13	13:1	3 Oct + m6↑	A4↑	4441	4400 / +41
14	14:1	3 Oct + m7↓	B♭4↓	4569	4600 / -31
15	15:1	3 Oct + M7	B4	4702	4700 / +2
16	16:1	4 Octaves	C5	4800	4800 / ± 0

Table 1: The first sixteen partials of the harmonic series, with intervals, ET approximations, and cent deviations. Negative cent deviation = flatter than ET; positive = sharper.

- **Common-practice triadic harmony** (fully discrete, fully functional): TSL uses triadic projections but liberates them from functional directionality.
- **Strict spectralism** (fully continuous, fully derived from a single root): TSL admits multiple roots simultaneously and allows discrete (modal) projections alongside spectral ones.
- **Pitch-class set theory** (fully discrete, non-acoustic): TSL grounds every pitch selection in acoustic justification.

The system’s intellectual lineage includes Debussy’s parallelism and whole-tone superposition, Ravel’s polytonal layering, Messiaen’s **modes of limited transposition** and chord-as-color philosophy (Messiaen 1944), Hindemith’s tonal-gravity series (Hindemith 1945), and most directly the spectral analyses and compositional procedures of Gérard Grisey and Tristan Murail (Grisey 1987; Murail 2005).

Scope and Structure of This Study

Section 2 establishes the acoustic foundations rigorously, with numerical tables and precise interval analysis. Section 3 develops the formal machinery of TSL: the projection operation, its algebra, and the compositional algorithm derived from it. Section 4 applies the formalism to concrete analytical case studies. Sections 5 and 6 extend the system into modal and microtonal territory respectively. Section 7 situates TSL historically through analytical (not biographical) engagement with key precursors. Section 8 articulates the full compositional methodology step by step. Section 9 provides extended worked examples. Every section connects theory to composition: no concept is introduced without showing its musical consequence.

ACOUSTIC FOUNDATIONS: THE HARMONIC SERIES AND ITS COMPOSITIONAL IMPLICATIONS

The Physics of Vibrating Strings and Columns

A string or air column of length L vibrating at fundamental frequency f_1 produces, as a consequence of the wave equation, an infinite series of modes at integer multiples of f_1 :

$$f_n = n \cdot f_1, \quad n \in \{1, 2, 3, 4, \dots\}$$

These are the **natural harmonics** or **partials** of the tone. Their amplitudes diminish with n (at a rate determined by the excitation mechanism and resonator geometry), but they are always present; every musical tone is, in reality, a superposition of this series. The composer who wishes to write spectrally coherent harmony must therefore think of a pitch not as a point but as a **column of overtones** — a vertical structure extending upward in pitch-space from the fundamental, with decreasing energy but increasing harmonic complexity.

The Overtone Series: Frequency Ratios and Interval Derivation

Taking $f_1 = 1$ (normalized), the first sixteen partials produce the following frequency ratios, intervals above the fundamental, and equal-temperament approximations:

Several observations are immediately compositionally relevant:

Partial 5 (major third): The just major third (5:4 ratio) is 14 cents flat relative to equal temperament. This is the most musically significant deviation: the ET major third is perceptibly sharper than the natural harmonic, contributing to the slight roughness characteristic of ET harmony. **Partial 7 (natural seventh):** At 31 cents below the ET minor seventh (B♭), this partial has no good ET approximation. It is the “blue note” of jazz, the septimal minor seventh of extended just intonation. Ignoring it entirely — as ET harmony does — discards a significant resource. **Partial 11 (eleventh harmonic):** 51 cents above F# (or equivalently 49 cents below G), this lies almost exactly midway between F# and G — a quarter-tone. It appears prominently in Grisey’s **Partiels** as a structural pitch (Grisey 1985). **Partial 13**

Partial	Interval	Deviation	Compositional Note
11	aug4↑	+51¢	Quarter-tone; no ET approximation
13	m6↑	+41¢	Near quarter-tone; must be notated microtonally
7	m7↓	−31¢	Septimal seventh; bluesy, primal quality
14	m7↓	−31¢	Same ratio as partial 7, octave higher
5	M3↓	−14¢	ET major third is slightly sharp
10	M3↓	−14¢	Same ratio as partial 5, octave higher
9	M2↑	+4¢	Nearly in ET; safe to approximate
3	P5↑	+2¢	Nearly in ET; safe to approximate
6	P5↑	+2¢	Same ratio as partial 3, octave higher
12	P5↑	+2¢	Same ratio as partial 3, two octaves higher
15	M7↑	+2¢	Nearly in ET
2	Oct	0¢	Pure octave; exact in ET

Table 2: Partially sorted by cent deviation magnitude. High-deviation partials are candidates for microtonal inflection in TSL.

(thirteenth harmonic): 41 cents above A (between A and A#), another near-quarter-tone with no ET equivalent.

Emergence of the Major Triad from the Overtone Series

The major triad emerges from the harmonic series as the earliest non-octave interval class grouping. Taking C1 as fundamental:

- Partial 2: C (octave) — reinforces root
- Partial 3: G (perfect fifth above C2) — first new pitch class
- Partial 4: C (two octaves) — reinforces root
- Partial 5: E (major third above C3) — second new pitch class

Thus {C, E, G} — the C major triad — is “written into” the physics of the C string by partials 4, 5, and 6 in the relationship 4:5:6. This is the **acoustic justification** for the triad’s privileged position in Western harmony: it is not a cultural convention but a mathematical consequence of vibrating systems. The ratio 4:5:6 in just intonation corresponds to:

- 4:5 = major third = 386.3 cents (vs. ET 400 cents, deviation −13.7¢)
- 5:6 = minor third = 315.6 cents (vs. ET 300 cents, deviation +15.6¢)
- 4:6 = perfect fifth = 702.0 cents (vs. ET 700 cents, deviation +2.0¢)

The minor triad does **not** emerge directly from a single harmonic series; it requires either a subharmonic series (acoustically real but weaker in most instruments), or construction from ratios 10:12:15, which involves a more complex arithmetic. This asymmetry between major and minor triads — one a natural resonance pattern, the other a constructed approximation — has consequences for TSL: **major projections carry greater acoustic weight than minor projections** when working from a single generator.

Deviation from Equal Temperament: Compositional Implications

The cent-deviation column of Table 1 reveals a crucial pattern: **equal temperament is a compromise tuning** that introduces systematic errors in every interval except the octave. The composer working with TSL faces a choice: **Option A — Equal**

Temperament Approximation: Accept ET deviations and work within the twelve-pitch system. This permits conventional notation and performance on standard instruments, but introduces systematic roughness into spectral harmonies. **Option B — Just Intonation:** Tune pitches to their exact harmonic ratios. Spectrally pure but requires either electronic realization or specially trained performers; notation requires microtonal accidentals. **Option C — Spectral Smoothing:** Use ET for structural pitches (low partials), introduce microtonal inflection for coloristic partials (high partials). This is the approach taken by Grisey and Murail in practice (Grisey 1987; Murail 2005), and is the approach recommended within TSL for acoustic instruments. The following table summarizes the cent deviations of the first twelve partials, sorted by magnitude, for rapid compositional assessment:

The Harmonic Series on C

FORMALIZATION OF TRIADIC SPECTRAL LAYERING

This section develops the core formalism of TSL. The goal is a set of operations that are precise enough to guide compositional decisions and flexible enough to admit a wide range of musical outcomes.

Basic Definitions

Definition 3.1 (Pitch Space). Let $\mathcal{P}$ denote the continuous pitch space parameterized by frequency in cents above a reference pitch (conventionally C0 = 16.35 Hz). Equal-temperament pitches are the lattice points $\{0, 100, 200, \dots\} \subset \mathcal{P}$, but $\mathcal{P}$ is continuous and admits microtonal pitches at arbitrary cent values. **Definition 3.2 (Chord).** A **chord** is a finite multiset $C = \{p_1, p_2, \dots, p_n\} \subset \mathcal{P}$ of pitches with associated amplitudes (or, in the discrete compositional setting, voice roles). **Definition 3.3 (Projection Set).** For a pitch $p \in \mathcal{P}$, the **spectral projection** $S(p)$ is the ordered sequence of overtone partials:

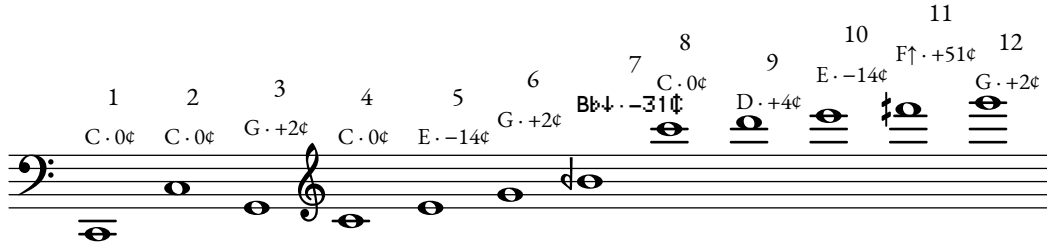


Figure 1: Harmonic series on C (partials 1–12) with cent deviations annotated above each note. Partial 7 (B $\flat$, -31¢) shown with half-flat accidental; partial 11 (F $\sharp$, +51¢) with quarter-sharp accidental.

$$S(p) = \{p + 1200 \cdot \log_2(n) : n \in \{1, 2, 3, \dots, N\}\}$$

where N is a chosen cutoff partial. The **triadic projection** $T(p)$ is the restriction of $S(p)$ to the partials that, when reduced modulo the octave and approximated to the nearest ET pitch, form a major triad on p :

$$T(p) = \{p, p + 400, p + 700\} \pmod{1200}$$

(i.e., the pitch class set {root, major third, perfect fifth} of p).

Definition 3.4 (Modal Projection). The **modal projection** $M(p, \Lambda)$ of pitch p under mode Λ is the set of pitches in Λ that lie within a chosen radius of a partial of $S(p)$:

$$M(p, \Lambda) = \{q \in \Lambda : \exists n \in \mathbb{N}, |q - S_{n(p)}| < \varepsilon\}$$

where ε is a tolerance parameter (typically 50–100 cents for ET modes, smaller for just-intonation modes). **Definition 3.5 (TSL Field).** Given a chord $C = \{p_1, \dots, p_n\}$, the **Triadic Spectral Layering field** $\Phi(C)$ is:

$$\Phi(C) = \bigcup_{i=1}^{\{n\}} T(p_i)$$

The field $\Phi(C)$ is the union of all triadic projections of the constituent pitches. It is the raw pitch collection from which the composer selects, reduces, and voices the realized harmony.

The Five Core Operations

Operation 1: Projection

Projection maps each pitch of C to its triadic (or spectral, or modal) projection set. The result is an expanded pitch collection — typically 9 to 30 pitches depending on the number of generators and projection type. For a triad $C = \{p_1, p_2, p_3\}$ with triadic projection:

$$\Phi(C) = T(p_1) \cup T(p_2) \cup T(p_3)$$

This generates at most 9 pitch classes (3 triads $\times$ 3 notes), often fewer due to overlaps.

Operation 2: Union and Intersection

The **union** $T(p_1) \cup T(p_2) \cup \dots$ yields the maximally dense pitch set — every pitch implied by any generator. The **intersection** $T(p_1) \cap T(p_2) \cap \dots$ yields only pitches shared by all projections — the points of strongest acoustic consensus. In practice, intersections between non-related triads are often empty or contain only the octave; they function as anchors around which a reduction is organized. **Weighted union** assigns a coefficient $w_i \in [0, 1]$ to each projection $T(p_i)$, where w_i reflects the acoustic salience of p_i within the chord (typically: bass pitch $w = 1.0$, middle voices $w = 0.7$, upper voices $w = 0.5$).

Operation 3: Reduction

Reduction maps the full field $\Phi(C)$ to a performable subset $R \subset \Phi(C)$ of cardinality k , where k is determined by:

- Ensemble size / number of available voices
- Desired harmonic density (transparent, medium, dense)
- Voice-leading requirements from the preceding harmony

Reduction criteria (applied in order of priority):

1. **Retain all pitches with $w \geq 0.8$** (high-salience fundamentals and fifths)
2. **Retain pitches shared by two or more projections** (consensus tones)
3. **Retain at least one characteristic pitch per projection** (color representation)
4. **Eliminate octave duplicates except in bass-register doublings**
5. **Fill remaining slots with highest-weighted remaining pitches**

Operation 4: Weighting

Within the field, each pitch $q \in \Phi(C)$ receives a **total weight**:

$$W(q) = \sum_{i: q \in T(p_i)} w_i \cdot \frac{1}{n(q, p_i)}$$

where $n(q, p_i)$ is the partial number of q relative to generator p_i (lower partials receive higher weight). This number guides orchestration: high-weight pitches are given to prominent instruments, sustained in the bass, or doubled; low-weight pitches are given to high winds, fluttering strings, or short articulations.

Operation 5: Register Distribution

Pitch Class	Source(s)	Partial Role	Total Weight	Interval Function
C	$T(C4)$	Root	1.00	Root of C minor; fundamental
D	$T(G4)$	Fifth	0.64	Major second above C
E	$T(C4)$	M3	0.70	Major seventh of field
E $\flat$	$T(E\flat4)$	Root	0.70	Minor third of C minor
G	$T(C4), T(E\flat4), T(G4)$	Fifth/Root	2.10	Consensus tone ; strongest
B $\flat$	$T(E\flat4)$	Fifth	0.56	Minor seventh of C
B	$T(G4)$	M3	0.56	Major seventh of C

Table 3: TSL field of C minor: pitch classes, generating sources, and weights.

Register distribution assigns each pitch in the reduction R to a specific octave based on:

- **Bass zone** (below C3): fundamentals, strong fifths only
- **Tenor zone** (C3–C4): roots and fifths, possibly thirds
- **Alto zone** (C4–C5): thirds, sevenths, modal color tones
- **Soprano zone** (C5 and above): high partials, microtonal inflections, spectral shimmer

The rule is: **weight increases downward** (heavier harmonics in the bass), **spectral complexity increases upward** (more remote partials in the treble). This mirrors the physics of orchestral resonance.

The Compositional Algorithm

The following algorithm summarizes TSL from initial chord choice to voiced harmonic field: **Step 1 — Choose Generator Chord C** : Select the base pitches from which projections will be generated. This may be any chord: a traditional triad, an interval dyad, a quartal stack, or a freely chosen pitch collection. **Step 2 — Select Projection Type**: Choose triadic, spectral (full overtone series to partial N), or modal projection $M(p, \Lambda)$ for each generator pitch. Different generators within the same chord may use different projection types. **Step 3 — Compute Field $\Phi(C)$** : Apply projection to each generator; take the union (weighted or unweighted). **Step 4 — Assign Weights $W(q)$** : Compute weights for each field pitch based on partial number and generator weight. **Step 5 — Reduce to Target Density k** : Apply reduction criteria to select k pitches from $\Phi(C)$. **Step 6 — Distribute in Register**: Assign pitches to register zones, choosing octave for each pitch class. **Step 7 — Voice**: Write the realized chord with specific voice assignments, instrumental doublings, dynamics, and articulations consistent with the weight map. **Step 8 — Voice-Lead to Next Field**: Treat the next harmony as a new field $\Phi(C')$ and minimize displacement (in pitch-space cents) between R and R' , subject to the constraint that characteristic pitches of $\Phi(C')$ are represented. **Step 9 — Introduce Microtonal Inflection** (optional): Shade any pitch in R by its cent deviation from ET (see Table 2) to increase spectral purity.

ANALYTICAL CASE STUDIES

Case Study A: C Minor as Generator

Step 1: The Generator Chord

Let $C = \{C4, E\flat4, G4\}$ — C minor triad in close position. We treat this as three pitch generators:

- $p_1 = C4$ (root; weight $w_1 = 1.0$)
- $p_2 = E\flat4$ (minor third; weight $w_2 = 0.7$)
- $p_3 = G4$ (fifth; weight $w_3 = 0.8$)

Step 2: Triadic Projections

Each generator produces a major triad projection:

- $T(C4) = \{C, E, G\}$ — C major
- $T(E\flat4) = \{E\flat, G, B\flat\}$ — E $\flat$ major
- $T(G4) = \{G, B, D\}$ — G major

Step 3: Full Field

$$\Phi(\{C_{\min}\}) = \{C, E\flat, E, G, B\flat, B, D\}$$

This is a seven-pitch collection (pitch classes). Let us examine its content:

Key observation: G is the consensus tone (weight 2.10), appearing in all three projections. This makes it the structural anchor of any voicing derived from this field. The field contains both E $\flat$ and E natural, and both B $\flat$ and B natural — chromatic frictions that provide harmonic richness without being arbitrary (each pitch is acoustically grounded).

Step 4: Reductions

Reduction A — Dense Cluster (all 7 pitches): Retain the entire field. This creates a highly complex chromatic sonority.

Reduction B — Transparent (4 pitches, highest weights): Select: G (2.10), C (1.00), E (0.70), D (0.64) → yields C major add9, no third — an open, luminous sound.

Reduction C — Dark Chromatic (5 pitches): Select: G (2.10), C (1.00), E $\flat$ (0.70), B $\flat$ (0.56), B (0.56) → a minor seventh chord with major seventh added — dense and ambiguous.

Reduction D — Spectral Blend (6 pitches, excluding lowest-weight): Select: G, C, E, E $\flat$, D, B $\flat$ → excludes B; retains the chromatic E $\flat$ /E pair within a major-ninth context.

Step 5: Voicings

Pianistic Voicing (Reduction B — Transparent)

LH: C2 (octave), G2 RH: D4, G4, E5 This places the highest-weight pitch (G) in both hands, uses the bass octave for C, and distributes D and E in the treble. The result is an open, quartal-flavored sound with spectral justification.

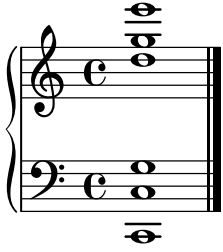


Figure 2: TSL transparent reduction of the C minor field. Piano grand staff: bass shows {C–C–G}, treble shows {D–G–E}.
Weight map: G 2.10 · C 1.00 · E 0.70 · D 0.64.

Orchestral Voicing (Reduction C — Dark Chromatic)

Pitches: {C, E $\flat$, G, B $\flat$, B} distributed as:

- Double basses / celli: C2
- Violas / celli: E $\flat$ 3, G3
- Violins II: B $\flat$ 4, B4 (simultaneous half-step — maximum tension point)
- Violins I: G5 (overtone doubling; stabilizes the field)

The semitone B $\flat$ /B in the upper strings creates a spectral “roughness zone” characteristic of late Grisey — not arbitrary dissonance but a consequence of superimposed projections.

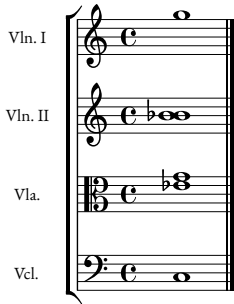


Figure 3: TSL dark chromatic orchestral voicing of the C minor field. Four string staves: Vcl. C2 · Vla. {E $\flat$ 3, G3} · Vln. II {B $\flat$ 4, B4} (spectral friction semitone) · Vln. I G5 (overtone doubling).

Case Study B: Dominant Seventh as Spectral Object

The dominant seventh chord V7 — e.g., G7 = {G, B, D, F} — is well known as a functional harmony driving to the tonic. TSL reveals it as an acoustically motivated chord: G, B, and D form a G major triad (partials 4:5:6 of G), while F is close to (but not identical with) the 7th partial of G (which would be F –31¢). This is not coincidence but the historical reason why V7 sounds “incomplete”: the ear expects the F to resolve to the pure 7th harmonic, which is 31 cents lower. The conventional resolution V7→I is, in TSL terms, a spectral resolution — the dissonant partial (F, which clashes with the natural harmonic series of G)

resolves to the tonic C, where it becomes a fourth partial (C = fundamental, re-establishing the series). **TSL analysis of G7:** Generators and projections:

- $T(G) = \{G, B, D\}$ (G major)
- $T(B) = \{B, D\#, F\# \}$ (B major)
- $T(D) = \{D, F\#, A\}$ (D major)
- $T(F) = \{F, A, C\}$ (F major)

Full field:

$$\Phi(\{G7\}) = \{G, B, D, F, D\#, F\#, A, C\}$$

This eight-pitch collection is almost entirely chromatic: it contains all twelve pitch classes except E, E $\flat$ (=D $\sharp$ is present), A $\flat$, and B $\flat$. The weight map concentrates on G (appears in $T(G)$), D (appears in $T(G)$ and $T(D)$), and F $\sharp$ (appears in $T(B)$ and $T(D)$). A transparent reduction {G, D, F $\sharp$, A} produces a G major add6 — a quartal/add6 sound — rather than the conventional G dominant seventh. A darker reduction {G, B, D $\sharp$, F} produces an augmented major seventh sonority. Both are acoustically grounded by the same generator chord.

MODAL EXPANSION AS MULTI-SOURCE HARMONIC GENERATION

Modal Projection: Definition and Procedure

In Section 3, we defined modal projection $M(p, \Lambda)$ as the subset of a mode Λ that falls near the partials of $S(p)$. This extends TSL in a crucial direction: instead of generating only major triads from each generator, we can generate **entire modal fields**, selecting which pitches of a mode are “activated” by each generator’s overtone resonance. The procedure is:

1. Choose a modal collection Λ (e.g., Dorian, Lydian, whole-tone).
2. For each generator p_i , compute the overtone series $S(p_i)$.
3. Find which pitches $q \in \Lambda$ lie within ε cents of any partial $S_{n(p_i)}$.
4. These pitches form $M(p_i, \Lambda)$ — the modal projection of p_i .
5. The TSL modal field is $\cup_i M(p_i, \Lambda)$.

The result is a **differentially activated mode**: not every pitch of Λ is equally implied, but each is grounded in the overtone resonance of at least one generator. This provides acoustic justification for modal pitch selection that strict modal composition lacks.

Illustrative Example: C Dorian with Multiple Sources

Let $\Lambda = \text{C Dorian} = \{C, D, E\flat, F, G, A, B\flat\}$. Generator chord: {C, G} (a fifth, the most resonant two-note combination). Partial matching (tolerance $\varepsilon = 50\text{¢}$): **From C:**

- Partial 2 → C (C $\in \Lambda$ ✓)
- Partial 3 → G (G $\in \Lambda$ ✓)
- Partial 4 → C (already counted)
- Partial 5 → E (not in Λ — E $\flat$ is 100¢ away, outside ε ; not matched)
- Partial 6 → G (counted)
- Partial 7 → B $\flat$ (B $\flat$ $\in \Lambda$, deviation –31¢ < 50¢ ✓)

- Partial 9 $\rightarrow$ D ($D \in \Lambda \checkmark$)

$M(C, \text{Dorian}) = \{C, G, B\flat, D\}$ **From G:**

- Partial 2 $\rightarrow$ G ($G \in \Lambda \checkmark$)
- Partial 3 $\rightarrow$ D ($D \in \Lambda \checkmark$)
- Partial 4 $\rightarrow$ G (counted)
- Partial 5 $\rightarrow$ B (not in Λ — $B\flat$ is 100¢ away; not matched)
- Partial 6 $\rightarrow$ D (counted)
- Partial 7 $\rightarrow$ F ($F \in \Lambda$, $B\flat$ of G would be F — deviation of G's 7th partial from F: $969\text{¢} - 1000\text{¢} = -31\text{¢} \checkmark$)

Actually let me be more precise. The 7th partial of G ($G = 700\text{¢}$) lies at $700 + 3369/3 = \dots$ let me recalculate. G's partials above G (in cents above C):

- Partial 1: G (700¢ above C4, or in absolute terms: let $G4 = 0$ for calculation)
- Partial 2: $G5 = G + 1200\text{¢}$
- Partial 3: $D5 = G + 1902\text{¢} \rightarrow$ pitch class D
- Partial 4: $G6 = G + 2400\text{¢}$
- Partial 5: $B6 = G + 2786\text{¢} \rightarrow$ pitch class B (not in Λ)
- Partial 6: $D7 = G + 3102\text{¢} \rightarrow$ D
- Partial 7: $F7\downarrow = G + 3369\text{¢} \rightarrow$ pitch class F (F is 3400¢ above G in ET; deviation = $3369 - 3400 = -31\text{¢} < 50\text{¢} \checkmark$)
- Partial 9: $A = G + 3804\text{¢} \rightarrow$ pitch class A ($A \in \Lambda \checkmark$)

$M(G, \text{Dorian}) = \{G, D, F, A\}$ **Combined modal field:**

$$M(\{C, G\}, \text{Dorian}) = \{C, D, F, G, A, B\flat\}$$

Absent from the activated Dorian: only $E\flat$. The generator chord $\{C, G\}$ activates six of seven Dorian pitches by acoustic resonance; $E\flat$ is not implied by either generator's overtone series within the tolerance. This is compositionally significant: $E\flat$ can be used as a **non-resonant interjection** — a pitch that enters the texture from outside the acoustic field, creating a moment of modal friction.

Modal Interference: Coexistence of Chromatic Contradictions

One of TSL's most distinctive features is its principled admission of chromatically contradictory pitches — E and $E\flat$, B and $B\flat$, F and $F\sharp$ — within the same harmonic field. In strict functional harmony, such coexistences are dissonances requiring resolution. In pitch-class set theory, they are simply collections with high interval-class diversity. In TSL, they are **acoustically grounded modal interferences**: each pitch belongs to the projection of a different generator, and their coexistence reflects the simultaneous resonance of multiple sources. **Example: E and $E\flat$ in the C minor field (Section 4.1)**

- $E\flat \in T(E\flat4)$ — root of the $E\flat$ major projection, sourced from the minor-third generator.
- $E \in T(C4)$ — major third of the C major projection, sourced from the root generator.

The $E\flat/E$ pair is not a “clash” but an **acoustic chord** in which two generators each assert their third-partial major third. The listener perceives it not as wrong-note chromaticism but as timbral richness: the harmonic field is perceived as a unified resonance object rather than as two contradictory pitch-class assertions.

Perceptual Coherence: Why Fields Do Not Collapse into Noise

The perceptual question is legitimate: why does a 7- or 9-pitch field not collapse into undifferentiated harmonic noise? The answer lies in **spectral rootedness**. When a pitch collection contains strong acoustic consensus tones (pitches shared by multiple projections, appearing as low partials of the generators), the auditory system assigns these tones as perceptual roots and organizes the remaining pitches as spectral colorations of those roots. Psychoacoustic research (Terhardt 1974; Parncutt 1989) confirms that virtual pitch perception operates across complex textures: even in a 12-pitch cluster, listeners extract virtual fundamental frequencies that structure the percept. In TSL, the weight map (Section 3.2.3) makes this explicit: high-weight pitches (consensus tones, low-partial projections) function as perceptual roots; low-weight pitches (high-partial, single-projection tones) function as spectral coloration. The field is coherent because its weight structure mirrors the natural perceptual hierarchy of the harmonic series. Composers reinforce this coherence through:

- **Register:** heavy/high-weight pitches in the bass, light/low-weight in the treble.
- **Duration:** consensus tones sustained; spectral color tones short or ornamented.
- **Dynamics:** weight proportional to dynamic level within the ensemble.
- **Orchestration:** fundamental instruments (low strings, bass clarinet) carry the roots; high-partial instruments (flute harmonics, violin sul ponticello) carry the upper field.

MICROTONAL EXTENSION: FROM DISCRETE PITCHES TO CONTINUOUS SPECTRUM

The Motivating Problem

Equal temperament is a 12-point discretization of the continuous pitch axis. TSL, as developed so far, operates within this discretization: projections are computed in ET, deviations noted but not realized. This is pragmatically useful — it permits conventional notation and performance — but it sacrifices acoustic precision. The major third is 14¢ sharper than the natural harmonic; the septimal seventh is 31¢ flat; the eleventh partial has no ET equivalent at all. Microtonal extension addresses this problem by restoring **spectral accuracy** to the harmonic field: pitches in the reduction R may be inflected by their cent deviation from ET, creating a spectrally pure realization of the field.

Cent Deviations as Compositional Parameters

The cent-deviation column of Table 1 (Section 2.2) becomes, in the microtonal extension, a set of **compositional adjustments** applied to each pitch in the reduction:

- Every E in the field is lowered by 14¢ (ET major third corrected toward just).
- Every $B\flat$ generated by a septimal seventh projection is lowered by 31¢.
- Every $F\sharp$ generated by an 11th-partial projection is raised by 51¢ (effectively a quarter-tone).

Partial	Exact Deviation	Nearest Quarter-Tone	Notation
5	-14¢	0¢ (no adjustment)	Standard ♮ or use 1/8-tone down mark
7	-31¢	-50¢ (quarter-flat)	♭↓ or ↓ (half-flat symbol)
11	+51¢	+50¢ (quarter-sharp)	♯↑ or ↑ (half-sharp symbol)
13	+41¢	+50¢ (quarter-sharp)	♯↑ or ↑
14	-31¢	-50¢ (quarter-flat)	♭↓ or ↓

Table 4: Quarter-tone system as spectral approximation. Partial deviations rounded to nearest quarter-tone.

- Perfect fifths and octaves remain unchanged (deviations of $\pm 2\text{¢}$ and 0¢ respectively).

These adjustments require either:

1. **Electronic realization** (synthesizers, software instruments with microtonal tuning tables).
2. **Specially calibrated acoustic instruments** (strings, winds, voices — which can naturally produce any pitch; requires explicit cent markings in the score and trained performers).
3. **Quarter-tone notation** for the coarser inflections ($\pm 50\text{¢}$, accessible on standard microtonal instruments).

Neutral Intervals and the Quarter-Tone System

Between the semitone (100¢) and the whole tone (200¢) lies the **neutral second** (approximately 150¢), characteristic of Arabic maqam, Turkish makam, and other non-Western tonal systems. In TSL, neutral intervals arise naturally as the beat between adjacent projections: **Example:** The 11th partial of C lies at 551¢ ($F\sharp + 51\text{¢} \approx$ halfway between $F\sharp$ and G). The 3rd partial of G lies at 702¢ (G , nearly exact). These two partials are 151¢ apart — a neutral second. If both are present in a TSL field (as they often will be when C and G are generators), the composer may realize this as an explicit quarter-tone cluster, a glissando between them, or a vibrato encompassing both. The quarter-tone system (24 equal divisions of the octave, step size 50¢) provides a practical approximation to spectral microtonality:

Spectral Smoothing

Spectral smoothing is the process of replacing sharp boundaries between ET pitch classes with continuous or near-continuous microtonal transitions. In a TSL field, two adjacent pitches (e.g., $E\flat$ and E , 100¢ apart) can be smoothed by:

1. Assigning each to a slightly adjusted intonation that reduces their mutual beating.
2. Inserting a glissando or microtonal trill that scans the cent-space between them.
3. Distributing $E\flat$ to one instrumental group (slightly higher, toward $E\flat + 14\text{¢} = E\flat$ just major third of $B\flat$) and E to another (slightly lower, $E - 14\text{¢} = E$ just major third of C).

The result is a harmonic field in which the boundary between adjacent chromatic pitches is not a discrete edge but a gradient — a **spectral blur** that, paradoxically, increases the perceived tonal coherence of the field by eliminating the harsh beating of ET semitones.

Pitch Fields vs. Pitch Sets

The culmination of microtonal extension is the conceptual transition from **pitch set** (a finite collection of discrete pitch classes) to **pitch field** (a region of pitch-space with varying density and weight). In a pitch field:

- Every position in pitch-space has an associated **activation weight** $W(x)$, $x \in \mathcal{P}$.
- The weight function is high at exact partial positions, decreasing away from them.
- A realization selects a finite sample from this continuous function, but the **function itself** is the compositional object.

This is the approach of Grisey's **Espaces acoustiques** cycle (Grisey 1985): each movement is a realization of the pitch field of a single bass E, progressively higher partials being introduced as the cycle progresses. The field is always the same acoustic object; the compositions are different samplings of it. In TSL, the pitch field is more complex (multiple generators) but the same logic applies: the composer works with the field function, and realizations are samples. Different orchestrations of the same field are not different harmonies — they are different perspectives on the same acoustic object.

Microtonal Cluster from C Minor Field

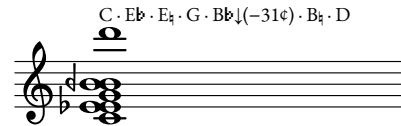


Figure 4: Microtonal TSL field of C minor. Seven-note spectral cluster: $C \cdot E\flat \cdot E\sharp \cdot G \cdot B\flat\downarrow(-31\text{¢}) \cdot B\sharp \cdot D$, notated as a single whole-note chord.

Microtonal Voice-Leading

When moving between two microtonal TSL fields, voice-leading operates on the cent-space rather than the semitone grid. Minimal voice-leading means minimizing the total displacement $\sum_i |q_i - q'_i|$ in cents, where q_i are the pitches of the first field and q'_i the pitches of the second. In microtonal space, what appears as a semitone shift (100¢) may in reality be a combination of a 14¢ intonation correction and a 86¢ pitch movement — structurally different from a pure 100¢ shift. The composer can exploit this: transitions between fields that seem harmonically disjunct in ET notation may in fact be acoustically smooth in microtonal space, if the inflections are managed appropriately.

HISTORICAL CONTEXT: ANALYTICAL PERSPECTIVES

Claude Debussy: Harmonic Fields through Parallelism

Debussy's harmonic innovation is not primarily about chord types (ninth chords, whole-tone chords, pentatonic formations) but about **harmonic behavior**: chords no longer progress functionally but **move in parallel**, maintaining their internal structure while translating in pitch-space. In "La cathédrale engloutie" (from *Préludes*, Book I), parallel ninth chords — {root, fifth, ninth, third, seventh} in consistent voicing — slide up and down the piano in motion that is modally free. This is, in TSL terms, a **constant-structure field translation**: the harmonic field of each chord has the same projection pattern, and the generator moves by step. The ear follows the generator (the bass line) and perceives the field as a timbral color rather than a functional harmony. Debussy's whole-tone harmonies achieve an even more radical effect: the whole-tone scale generates, under TSL projection, highly symmetric fields with no clear root (all generators are acoustically equivalent by the symmetry of the scale). This produces the characteristic Debussyan suspended quality — the sense of harmonic floating without resolution impulse. The mechanism in TSL terms: when all generators of a chord are related by equal intervals (as in the whole-tone scale or augmented triad), their projections overlap symmetrically, and the weight map has no single dominant consensus tone. The harmonic field is uniform — no pitch pulls stronger than another — and the result is tonal stasis.

Maurice Ravel: Controlled Color Layering

Ravel's harmony is distinguished by its **precision of control**: where Debussy floats freely, Ravel stacks projection layers with architectural calculation. The "Feria" movement of *Rapsodie espagnole*, the jazz-influenced harmonies of *L'enfant et les sortilèges*, and the orchestration of *Bolero* all demonstrate an explicit awareness of how individual orchestral colors interact as acoustically superimposed fields. In TSL terms, Ravel is doing **register-controlled projection weighting**: each orchestral layer carries a specific subset of the harmonic field, and the blend of layers is calibrated so that the resulting sound object has a specific spectral profile. The famous Ravel orchestral "glitter" — celesta, harp, *sul tasto* strings, muted brass — is a TSL distribution in which:

- The bass instruments carry the roots (weight ≥ 0.9)
- The middle strings carry the consensus tones and thirds
- The high woodwinds and celesta carry the high-partial projections (weight 0.3–0.5)

The *Boléro* ostinato harmony is a pedagogical case: a single chord (Cm or C major depending on the variation) is maintained throughout, but the **projection realization** changes with each orchestral iteration — different subsets of the field, different register distributions, different timbral weights — generating the sense of progressive transformation from a single harmonic object.

Olivier Messiaen: Chord as Color Object

Messiaen explicitly theorized harmony as **color** — not metaphorically but quasi-synesthetically, claiming specific visual color correspondences to specific chord types. This is, stripped of its synesthetic wrapping, a precisely TSL-compatible position: each chord has an intrinsic acoustic identity (its spectral field) that the composer perceives as a unified perceptual object. Messiaen's **modes of limited transposition** (MLT) (Messiaen 1944) are, from the TSL perspective, modal projection spaces with restricted symmetry: modes that, when projected through triadic or spectral operations, generate highly interconnected fields with maximum internal overlap. Mode 2 (octatonic), for instance, when used as Λ in the modal projection $M(p, \Lambda)$, generates fields in which generators separated by minor thirds produce identical projection sets — a consequence of the mode's four-fold transposition symmetry. Messiaen's **chord of resonance** (chord de la résonance) — which he constructs by stacking partials 8–16 of a fundamental — is the explicit TSL spectral projection, realized as a vertical chord in the treble register. He uses it as a quasi-tonic: a sonority of maximum resonance that, like the tonic in functional harmony, provides harmonic stability — but stability grounded in acoustics rather than in tonal grammar. His **Added-value chords** and **chord complexes** in the *Quartet for the End of Time* and *Turangalila-Symphonie* demonstrate TSL reduction in action: from a complex spectral field, a small subset of high-weight pitches is selected and repeated as a rhythmically distinctive unit, giving the chord both harmonic and rhythmic identity.

Gérard Grisey and Tristan Murail: Explicit Spectralism

The Spectral school, emerging from IRCAM in the 1970s, represents the most direct antecedent of TSL. Grisey and Murail both worked from the explicit recognition that the harmonic series is the generative structure of musical harmony — that tonality itself is a cultural formalization of acoustic facts — and that composition should begin from spectral analysis rather than from inherited scale and chord systems. Grisey's "*Partiels*" (1975) is the paradigmatic spectral composition (Grisey 1985). The entire harmonic language derives from a Fourier analysis of a low E on the trombone. The score opens with that E, then adds, one by one, its partials up to partial 16, creating a crescendo of spectral complexity before dissolving into noise-band sonorities in which the discrete pitches merge into continuous spectral regions. In TSL terms, *Partiels* is a single-generator TSL work: $|C| = 1$, all projections drawn from the spectral projection $S(E_2)$ with increasing partial cutoff N over the course of the piece. The novelty of TSL relative to strict spectralism is the multi-generator extension: TSL takes two or more pitches as generators, superimposes their spectral projections, and works with the resulting interference pattern. Murail's "*Gondwana*" (1980) introduces **harmonic modulation**: the harmonic field morphs continuously from the field of one generator to another, like a slow tidal shift (Murail 1983; 2005). This corresponds in TSL to a continuous transformation of the generator chord $C(t)$ over time, with the field $\Phi(C(t))$ changing accordingly. Murail explicitly uses cent-accurate microtonal notation throughout, realizing the spectral field without ET approximation. Both composers use what TSL formalizes as the **weight-map orchestration principle**: low-partial pitches are given to low-register, high-amplitude instruments (trombone, bass clarinet); high-partial pitches to

Generator Type	Example	Field Character
Perfect fifth	{C, G}	Open, luminous; strong consensus at G, C, D, B $\flat$
Major triad	{C, E, G}	Rich, warm; T(E) adds G $\sharp$ /A $\flat$ friction
Minor triad	{C, E $\flat$, G}	Dense chromatic; E $\flat$ /E and B $\flat$ /B pairs
Tritone	{C, F $\sharp$ }	Maximal chromatic friction; no consensus tones
Major seventh	{C, B}	High spectral tension; B projects D $\sharp$ /A $\sharp$
Minor ninth	{C, D $\flat$ }	Cluster field; extremely dense at low partials
Quartal chord	{C, F, B $\flat$ }	Lydian-flavored field; F projects A/C, B $\flat$ projects D/F

Table 5: Generator chord types and their TSL field characteristics.

high-register, low-amplitude instruments (flute harmonics, violin harmonics, percussion). The timbre of the ensemble **mimes** the timbre of a single acoustic source.

COMPOSITIONAL METHODOLOGY

This section presents the complete TSL compositional method as a sequential procedure. Each step is concrete, the musical rationale is given, and the range of choices available at each step is specified.

Step 1: Choose the Generator Chord

The generator chord $C = \{p_1, \dots, p_n\}$ is the starting point. It need not be an aesthetically “good” chord in any traditional sense; it is the acoustic seed from which the field will grow. **Considerations:**

- **Interval structure of C :** fifths and fourths generate highly resonant fields with clear consensus tones; seconds and tritones generate denser, more chromatic fields with distributed weight.
- **Number of generators n :** 2–4 generators are ideal for manageable fields. More than 5 generators typically saturates the field (too many pitches, no clear weight hierarchy).
- **Register of C :** low-register generators produce denser, lower-pitched fields; high-register generators produce sparser, higher-pitched fields. The bass pitch always dominates the field.

Recommended starting generators for different harmonic characters:

Step 2: Select Projection Type

For each generator p_i , choose: **Option A — Triadic Projection** $T(p_i) = \{p_i, p_i + 4, p_i + 7\}$ (semitones): simplest; generates only major triad. Good for clear, tonal-adjacent fields. **Option B — Extended Triadic Projection:** include the seventh partial — $T_7(p_i) = \{p_i, p_i + 4, p_i + 7, p_i + 10(-31\text{c})\}$. Adds a septimal minor seventh; useful for jazz-derived fields. **Option C — Spectral Projection to Partial N :** full overtone series up to partial N . $N = 6$ gives the major triad; $N = 8$ adds the minor seventh and upper octave; $N = 16$ generates a rich chromatic field. Use for spectralist textures. **Option D — Modal Projection** $M(p_i, \Lambda)$: for fields grounded in a specific modal color. Choose Λ first, then derive which modal pitches each generator activates. Different generators within the same chord may use different projection types — e.g., the bass uses spectral projection

while upper voices use modal projection. This creates a **hybrid field** with spectral grounding in the bass and modal color in the treble.

Step 3: Compute the Field

Apply the chosen projection to each generator and compute the union (with weights). Tabulate the result as in Section 4.1, Table 3: list each pitch, its sources, its partial roles, and its total weight. This tabulation is the **compositional score** of the harmony: it contains all information needed for reduction, orchestration, and voice-leading.

Step 4: Assign Weights

Use the formula of Section 3.2.3:

$$W(q) = \sum_{\{i: q \in T(p_i)\}} \frac{w_i}{n(q, p_i)}$$

where w_i is the generator weight (bass = 1.0, middle voices = 0.7, upper voices = 0.5) and $n(q, p_i)$ is the partial number (root = 1, fifth = 3, major third = 5, etc.). Sort pitches by weight descending. The top three or four pitches will typically form the structural core of any reduction.

Step 5: Reduce to Target Density

Choose k (number of realized pitches) based on:

- **Ensemble:** string quartet → 4 pitches; full orchestra → up to 12; solo piano → 5–8.
- **Texture:** sparse → $k = 3$ –4; medium → $k = 5$ –7; dense → $k = 8$ –12.
- **Position in phrase:** climactic moments → denser; structural pillars → sparser.

Apply reduction criteria (Section 3.2.3): retain high-weight, consensus, and color pitches; eliminate redundant octave doublings.

Step 6: Distribute in Register

Assign each pitch in the reduction to a specific octave using the weight-register rule:

- Weight ≥ 1.0 : bass zone (C1–C3) or tenor zone foundation
- Weight 0.7–1.0: tenor/alto zone (C3–C5)
- Weight 0.4–0.7: alto/soprano zone (C4–C6)
- Weight < 0.4 : soprano zone (C5+) or harmonics/sul ponticello territory

The **bass pitch** is always the lowest generator (not necessarily the pitch with highest weight). If the generator chord is $\{C, E\flat, G\}$, the bass zone belongs to C regardless of weight: this is the spectral root.

Step 7: Voice the Chord

Assign specific pitches to specific instruments or voices. Rules:

1. The **spectral root** (lowest generator) must be present in the bass or it will be implied by the ear (which may be desirable or not).
2. Doubled pitches must be at octave intervals, not unisons in the same register (unless a unison blend is desired for timbre).
3. High-weight pitches may be doubled across octaves to reinforce their resonance.
4. Low-weight pitches should appear in single, high-register instruments (flute, violin harmonics) with short articulation.

Step 8: Voice-Lead Between Fields

To connect field $\Phi(C)$ to field $\Phi(C')$:

1. Compute the minimal bijection between the pitches of R and R' (in cents) — the assignment that minimizes total displacement.
2. For each voice, determine whether the motion is a common tone (0¢ displacement), a step (≤ 200 ¢), or a leap (> 200 ¢).
3. Maximize common tones and steps; assign leaps to the instruments with greatest mobility (solo winds, high strings).
4. If a pitch is a consensus tone in both fields (appears in both weight tables with high weight), treat it as a structural anchor: it should be sustained or returned quickly.

Microtonal voice-leading: when inflections are used, the voice-leading map operates in cent-space. A voice moving from E (−14¢ inflection) to $E\flat$ (0¢) travels 114¢ in just space — slightly more than a semitone — which sounds like a smooth chromatic step.

Step 9: Introduce Microtonal Variation

Apply cent adjustments as in Table 2 (Section 2.4) selectively:

- Always inflect septimal seventh partials (−31¢) when they appear.
- Inflect major thirds (−14¢) in solo contexts where the slight sharpness of ET is noticeable.
- Use quarter-tone inflections (+51¢ for 11th partial, +41¢ for 13th) in textures that support microtonality.
- In ensemble contexts, notate inflections with arrow accidentals and provide a cent legend at the beginning of the score.

EXTENDED WORKED EXAMPLES

Example 1: A Short Two-Field Progression (String Quartet)

Setup: Two sequential harmonic fields for string quartet, connected by smooth voice-leading. **Field 1: Generator** $\{E\flat, B\flat\}$ — a perfect fifth. Triadic projection:

- $T(E\flat) = \{E\flat, G, B\flat\}$
- $T(B\flat) = \{B\flat, D, F\}$

Field 1: $\Phi_1 = \{E\flat, G, B\flat, D, F\}$ — a $B\flat$ major ninth sonority. Weights:

- $B\flat$: appears in both $\rightarrow W = 1.0 + 0.5 = 1.5$ (root of $T(B\flat)$, fifth of $T(E\flat)$)
- $E\flat$: root of $T(E\flat) \rightarrow W = 1.0$
- G : M3 of $T(E\flat) \rightarrow W = 0.5$
- D : M3 of $T(B\flat) \rightarrow W = 0.35$
- F : P5 of $T(B\flat) \rightarrow W = 0.5$

Reduction ($k = 4$): $\{B\flat, E\flat, G, F\}$ (weights 1.5, 1.0, 0.5, 0.5)

Voicing for string quartet:

- Cello: $E\flat 2$
- Viola: $B\flat 3$
- Violin II: $G 4$
- Violin I: $F 5$ (a ninth above $E\flat$ — spectral tension)

Field 2: Generator $\{F, C\}$ — another perfect fifth, a fifth higher.

Triadic projection:

- $T(F) = \{F, A, C\}$
- $T(C) = \{C, E, G\}$

Field 2: $\Phi_2 = \{F, A, C, E, G\}$ — C major ninth sonority. Weights:

- C : root of $T(C)$, fifth of $T(F) \rightarrow W = 1.5$
- F : root of $T(F) \rightarrow W = 1.0$
- G : P5 of $T(C) \rightarrow W = 0.5$
- A : M3 of $T(F) \rightarrow W = 0.5$
- E : M3 of $T(C) \rightarrow W = 0.35$

Reduction ($k = 4$): $\{C, F, G, A\}$ Voicing for string quartet:

- Cello: $C 2$
- Viola: $G 3$ (common tone with Field 1!)
- Violin II: $A 4$ (moves from $G 4$ by one step up)
- Violin I: $F 5$ (common tone with Field 1!)

Voice-leading analysis:

- Cello: $E\flat 2 \rightarrow C 2$ (descending minor third, 300¢)
- Viola: $B\flat 3 \rightarrow G 3$ (descending minor third, 300¢ — parallel motion with cello)
- Violin II: $G 4 \rightarrow A 4$ (ascending major second, 200¢)
- Violin I: $F 5 \rightarrow F 5$ (common tone, 0¢)

Total displacement: $300 + 300 + 200 + 0 = 800$ ¢ — efficient two-field motion.

Example 2: Evolving Harmonic Texture — A Spectral Crescendo

This example demonstrates a TSL field expanding progressively, beginning from a single generator and adding projection layers. This models Grisey's **Partiels** strategy within a multi-generator TSL framework. **Phrase Architecture:** 4 stages, each a separate harmonic moment. The generator chord expands by one pitch per stage. **Stage 1: Single generator** — $C_1 = \{E2\}$ Spectral projection to partial 6: $\{E2, E3, B3, E4, G\sharp 4, B4\}$ This is the E major chord in register (partials 1–6). Very pure, rooted, consonant. **Stage 2: Add second generator** — $C_2 = \{E2, B2\}$ $T(E2) \cup T(B2) = \{E, G\sharp, B\} \cup \{B, D\sharp, F\sharp\}$ Field: $\{E, G\sharp, B, D\sharp, F\sharp\}$ — E major ninth with major seventh. **Stage 3: Add third generator** — $C_3 = \{E2, B2, D3\}$ $T(D3) = \{D, F\sharp, A\}$ New pitch: A ($F\sharp$ was already present; D is a new color in the field) Field: $\{E, G\sharp, B, D\sharp, F\sharp, A, D\}$ — seven pitches; an extremely rich sonority containing both $D\sharp$ and D, both A and $G\sharp$ (one

Figure 5 shows a musical score for a string quartet with four staves: Vln. I, Vln. II, Vla., and Vcl. The score is divided into two measures. Above the staves, two pitch classes are indicated: F5 and (common tone). The notes in the staves are: Vln. I (F5), Vln. II (G4), Vla. (Bb3), Vcl. (Eb2) in the first measure, and Vln. I (F5), Vln. II (A4 (+200¢)), Vla. (G3 (-300¢)), Vcl. (C2 (-300¢)) in the second measure.

Figure 5: TSL two-field string quartet progression. Field 1 (gen. $\{E\flat, B\flat\}$): Vcl. $E\flat 2 \cdot$ Vla. $B\flat 3 \cdot$ Vln. II $G 4 \cdot$ Vln. I $F 5$. Field 2 (gen. $\{F, C\}$): Vcl. $C 2 \cdot$ Vla. $G 3 \cdot$ Vln. II $A 4 \cdot$ Vln. I $F 5$. Total displacement: 800¢.

semitone separation each). **Stage 4: Add fourth generator with spectral 7th** — $C_4 = \{E2, B2, D3, G3(-31\text{¢})\} T_7(G3) = \{G, B, D, F(-31\text{¢})\}$ New pitches: G (already D, B present; G is new), $F\flat(-31\text{¢})$ = natural seventh of G. Field: $\{E, G\sharp, B, D\sharp, F\sharp, A, D, G, F(-31\text{¢})\}$ — nine pitches with a microtonal F. **Compositional result:** Over four stages, the harmony grows from an E major triad to a nine-pitch spectral cluster with a microtonal component. The crescendo of harmonic complexity mirrors a natural crescendo of dynamic and textural density.

Example 3: TSL Harmonic Sequence for Piano

A four-chord progression in which each chord is derived by a slight transformation of the generator: **Chord 1:** Generator $\{C, G\} \rightarrow$ field $\{C, E, G, D, B\flat\} \rightarrow$ reduction $\{C, G, E, D\}$ **Chord 2:** Generator $\{D, A\} \rightarrow$ field $\{D, F\sharp, A, E, C\} \rightarrow$ reduction $\{D, A, F\sharp, C\}$ **Chord 3:** Generator $\{E, B\} \rightarrow$ field $\{E, G\sharp, B, F\sharp, D\} \rightarrow$ reduction $\{E, B, G\sharp, D\}$ **Chord 4:** Generator $\{F\sharp, C\sharp\} \rightarrow$ field $\{F\sharp, A\sharp, C\sharp, G\sharp, E\} \rightarrow$ reduction $\{F\sharp, C\sharp, A\sharp, E\}$ Each chord generates a major ninth sonority (root, fifth, major third, major second from the fifth = major ninth above root). The generator moves upward by whole tone each time, producing a whole-tone sequence of TSL fields — a structural use of the whole-tone cycle within a spectrally grounded framework.

SYNTHESIS: TSL AS AN OPEN SYSTEM

Summary of the Method

Triadic Spectral Layering provides a compositional framework in which: **Harmony is generative:** every chord is derived from the acoustic projections of its constituent pitches, not from a pre-given harmonic vocabulary. **Harmony is weighted:** every pitch in a field has a quantified acoustic importance, guiding orchestration, register, and dynamics without recourse to arbitrary taste. **Harmony is reducible:** the full field is a resource to be sampled, not a mandatory sonority. The composer controls density through systematic reduction. **Harmony is continuous:** the step from equal temperament to microtonality is not a revolution but an extension — the cent deviations are already latent in the weight tables, ready to be realized. **Harmony is transformable:** voice-leading between fields operates by minimal displacement in pitch-space, creating smooth harmonic motion from arbitrary starting points.

Extensions and Open Problems

Several extensions of TSL remain to be developed: **Rhythmic fields:** Just as TSL generates pitch fields from generator chords, a temporal analogue would generate **rhythmic fields** from generator durations — applying the proportional logic of the overtone series to time. Messiaen's **personnages rythmiques** and Nancarrow's tempo canons suggest the direction. **Timbre as field:** If timbre is understood as a spectral field (the profile of partial amplitudes of a specific instrument), then timbre and harmony become the same phenomenon at different scales — a timbral blend is a harmonic field realized by spectral mixing. TSL can be extended to compute **timbral projections** analogous to pitch projections. **Dynamic TSL (field morphology):** Murail's

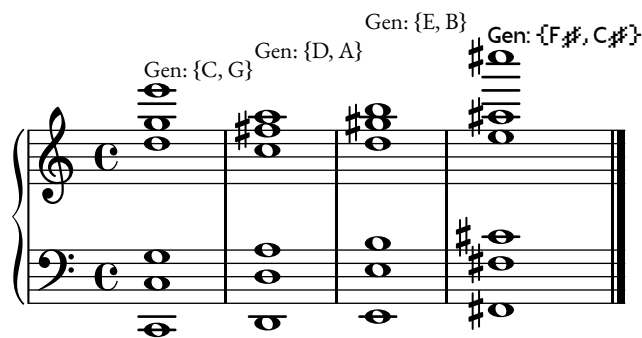


Figure 6: TSL whole-tone generator sequence, piano. Four measures; generators ascend by whole tone: $\{C,G\} \rightarrow \{D,A\} \rightarrow \{E,B\} \rightarrow \{F\#,C\#\}$. Each field voiced as major add9. Generator labels appear above each treble chord.

harmonic modulation — the continuous transformation of the field function $\Phi(C(t))$ over time — remains an open compositional challenge within TSL. Formal tools for describing field morphology (interpolation between projections, smooth weight transfer, gradual microtonal inflection) are needed. **Subharmonic projection:** TSL as developed projects upward (overtone series). A subharmonic projection would project downward, generating undertone series fields. These are acoustically weaker but perceptually real (virtual pitch phenomena), and they may generate the minor-triad projections that TSL currently lacks.

On Learning TSL

A composer wishing to internalize TSL as a working tool should proceed through the following stages of self-study: **Stage 1 — Acoustic internalization:** Sing or play the overtone series from multiple roots until the partial pitches and their ET deviations are aurally familiar. Use sine-tone generators or string harmonics. **Stage 2 — Projection practice:** For any triad, compute its TSL field (triadic projection only) without reference to tables. Check against the weight map. Do this for 20–30 chords spanning all twelve roots. **Stage 3 — Reduction practice:** From each field, produce three different reductions (sparse, medium, dense) and voice them for piano. Listen carefully to how the weight map manifests as perceived harmonic gravity. **Stage 4 — Voice-leading connections:** Connect pairs of fields by minimal voice-leading. Transcribe and analyze Grisey’s *Partiels* and Murail’s *Gondwana* using the TSL weight-map framework. **Stage 5 — Microtonal extension:** Realize several TSL fields with full spectral inflection, using software synthesis. Compare the ET and microtonal versions aurally. Then write for acoustic instruments with cent markings. **Stage 6 — Free composition:** Apply the full compositional algorithm to original work. Generate three or four fields, connect them, orchestrate them, and evaluate the result against the three criteria: acoustic coherence (does the weight map manifest in the sound?), harmonic richness (is the field sufficiently complex?), and compositional intentionality (does the field choice serve the expressive purpose?).

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